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Finite temperature effects on the induced Chern-Simons term in noncommutative geometry

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Abstract: The one-loop contribution to vacuum polarization is calculated for the adjoint fermions in three dimensional noncommutative spaces, both at zero and finite temperature. At zero temperature, we confirm a previously found result for the parity odd part and subsequently analyze the even parity part, which exhibits UV/IR mixing. We discuss in detail, two regimes of the high temperature behavior of the parity odd part. When the thermal wavelength is much smaller, as compared to the noncommutativity scale, we find an interesting Fermi-Bose transmutation in the nonplanar part.

Keywords: Chern-Simons Theories, Non-Commutative Geometry, Thermal Field Theory.

Contents

1. Introduction

The study of field theoretical models on noncommutative spaces has attracted considerable attention in recent times (for a review see, [1]), because of their natural appearance in certain limits of the string and M theories, in a constant background B field. The inverse of $|B|$ plays the role of the dimensionful noncommutativity paramater θ [2, 3]. Noncommutative theories show very interesting perturbative behavior [4, 5, 6, 7], due to the presence of an extra Moyal phase at the vertex. In a simple model, this phase has been shown to arise due to the interaction of pairs of oppositely charged particles, restricted to the lowest Landau level (LLL), by a strong magnetic field [8, 9]. Since the noncommutative field theory describes particles, which are extended like dipoles, it has been proposed that, the long distance behavior of the quantum Hall fluid [10] may be better modelled by a Chern-Simons (CS) theory [11, 12] in a noncommutative geometry [13, 14].

CS theories on noncommutative spaces are being extensively studied [15]–[22] due to their potential applications in condensed matter systems. A number of formal aspects of these theories, in parallel to their commutative counter parts, are also being investigated [23, 24]. It has been shown that the coefficient of the CS term remains quantized for the non-abelian and, surprisingly, also for the abelian theory, on a noncommutative space [25, 26]. Using the arguments of BRST invariance and linear vector supersymmetry, the Coleman-Hill theorem [27] has been extended to the noncommutative background, showing that the tree level CS term does not receive any more correction, beyond one loop, in pure CS gauge theories [28, 29].

The vacuum polarization tensor, at the one-loop level, is not affected by the noncommutative geometry, for the fermions in the fundamental representation. Charged fermions, in the adjoint representation induce a θ dependent CS term, which shows a discontinuous behavior in the noncommutativity parameter θ [30, 31].

In the ordinary CS theory, the effect of finite temperature, particularly on the parity odd sector [32], leads to a number of interesting results [33]. The parity odd part of the photon self-energy loses its analyticity at finite temperature [34, 35]. It has been pointed out that, invariance of the effective action under large gauge transformations, at non-zero temperatures, necessitates the incorporation of nonperturbative contributions [36]. Apart from these formal aspects, physical applications of the CS theories in noncommutative geometry, make it imperative to study these theories at finite temperature.

In this note, we study the effect of temperature on the induced CS term for the adjoint fermions. The parity odd part of the polarization tensor is analyzed carefully. Various limits, involving the noncommutativity paramater and the high and low temperature behaviors of the coefficient of the induced CS term are studied in detail and a number of interesting features pointed out. The analysis of the even parity sector reveals UV/IR mixing, a hall-mark of the noncommutative theories.

The paper is organized as follows. In section 2, we outline the general features of field theories on noncommutative spaces and then proceed to calculate the one-loop vacuum polarization tensor. Both the odd and even parity sectors are studied and the results contrasted with those of the ordinary CS theory. In section 3, the finite temperature effects are analyzed and various limiting behaviors are studied in detail. We conclude in section 4, after pointing out directions for future study.

2. Field theories on noncommutative spaces

There are two equivalent descriptions of the noncommutative spaces (for a lucid review, see [37]), which is taken as $2+1$ for our purposes. When the coordinates of the space are considered as operators, noncommutative R^3 is defined by the fundamental relation,

$$[X_i, X_j] = i\theta_{ij} \,, \tag{2.1}$$

where θ[1] is a constant antisymmetric matrix, carrying a dimension of the square of length. Hence, functions on R^3 would be functions of operators. However, a more convenient way is to work with functions of real variables, but with the ordinary product replaced by the Moyal star product. In this description, (2.1) takes the form,

$$[x_i, x_j]_{MB} = i\theta_{ij} \,, \tag{2.2}$$

where a Moyal bracket (MB) between two functions $f(x)$ and $g(x)$ is defined as,

$$[f(x), g(x)]_{MB} = (f * g)(x) - (g * f)(x) \,. \tag{2.3}$$

The associative star product is defined as,

$$(f * g)(x) = \left[\exp\left(\frac{i}{2}\theta_{\mu\nu}\partial_{\alpha_\mu}\partial_{\beta_\nu}\right) f(x+\alpha) g(x+\beta)\right]_{\alpha=\beta=0} . \tag{2.4}$$

[1]Henceforth, we only consider the case where $\theta_{0i} = 0$.

Notice that the star product, defined at a point, brings in nonlocality due to the presence of infinite number of derivatives. A great simplification, while carrying out perturbative expansions in a noncommutative field theory, occurs due to the following result:

$$\int (f * g)(x) d^3x = \int (g * f)(x) d^3x = \int f(x) g(x) d^3x \,. \tag{2.5}$$

When the interaction is switched off, the results of the ordinary theory go over to those of the noncommutative theory. The Green's functions are not modified and the perturbative calculations proceed along similar lines as those of the oridinary cases. However, the vertices carry additional phases originating from the Moyal bracket. Hence, at the quantum level, two types of diagrams need to be analyzed: planar and nonplanar. The planar diagrams are those, where the phase factor depends on the external momenta and the nonplanar diagrams are the ones, where the phase factor contains internal loop momentum. This is responsible for a number of interesting features of the noncommutative theories.

The QED action (for the construction of QED action and the Feynman rules, see [38]) for the two component fermionic field is given by,

$$S[A, m] = \int d^3x \left[-\frac{1}{4} F_{\mu\nu} * F^{\mu\nu} + (\overline{\psi} * (i\not{D} - m)\psi)(x) \right] , \tag{2.6}$$

where m is the bare mass of the fermion. The field strength tensor,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]_{MB} \,, \tag{2.7}$$

reveals that even in a U(1) theory, the gauge field A_μ is self-interacting, with the coupling constant g. It can be checked that the above action is invariant under the star gauge transformations[2] $\delta\psi = ig\alpha * \psi$ for the Dirac spinor (fundamental representation) and $\delta\psi = ig[\alpha, \psi]_{MB}$ for the Majorana spinor (adjoint representation). Here, the covariant derivative, for the Dirac fermion is $D_\mu\psi = \partial_\mu\psi - igA_\mu * \psi$ and for the Majorana spinors, it is given by, $D_\mu\psi = \partial_\mu\psi - ig[A_\mu, \psi]_{MB}$; in the limit $\theta \to 0$, the Majorana coupling to the gauge field vanishes.

For convenience of comparison with the existing results, we work in the euclidean space in 2+1 dimentions. The relevant Dirac gamma matrices, satisfying, $\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$, are given by, $\gamma^1 = i\sigma^2$, $\gamma^2 = i\sigma^3$, $\gamma^3 = i\sigma^1$, and some useful identities needed for later use are,

$$\begin{aligned} \gamma^\mu\gamma^\nu &= -\eta^{\mu\nu} - \epsilon^{\mu\nu\lambda}\gamma^\lambda \,, \qquad \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\lambda) = 2\epsilon^{\mu\nu\lambda} \,, \\ \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\rho) &= 2(\eta^{\mu\nu}\eta^{\rho\lambda} + \eta^{\mu\rho}\eta^{\nu\lambda} - \eta^{\mu\lambda}\eta^{\nu\rho}) \,. \end{aligned}$$

As has been mentioned earlier, the quadratic part of the action is the same as that of the ordinary theories and hence, the perturbative calculations can be carried out analogously. From the expansion of the action in the momentum space, it follows that, the vertex contains an additional phase factor $e^{i\tilde{p}k}$, as compared to the commutative case, here $\tilde{p}k = p_i\theta^{ij}k_j$.

[2] Although, the divergence of the current does not vanish (a consequence of the star product), it can be shown that a conserved charge does exist.

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2.1 Vacuum polarization in $2+1$ dimensions

In this section, we present the calculations for the vacuum polarization tensor, separating out the planar and nonplanar contributions. The odd and even parity sectors are discussed in appropriate subsections. The photon self-energy gets modified due to the presence of the Moyal phases at the vertices and is given by,

$$i\Pi^{\mu\nu}(p_3,\overline{p}) = -g^2 \int \frac{d^3k}{(2\pi)^3} \frac{M^{\mu\nu}}{((k-p)^2+m^2)(k^2+m^2)} \cdot (V)\,, \tag{2.8}$$

with,

$$M^{\mu\nu} = 2(2k^\mu k^\nu - (p^\mu k^\nu + p^\nu k^\mu) + \eta^{\mu\nu}(p.k - k^2) - m^2\eta^{\mu\nu} + mp^\alpha \epsilon^{\mu\nu\alpha})\,. \tag{2.9}$$

The only change in going to the noncommutative theory is the appearance of the Moyal phase V [39]; this turns out to be one, for the case of the Dirac fermions and $4\sin^2(\tilde{p}k/2)$ for the Majorana case. The dependence of V on the internal loop momentum implies that the polarization tensor can receive contributions from the non-planar diagrams as well.

The case of the Dirac fermions needs no further elaboration, as it reproduces the results of the ordinary theory; henceforth, we shall concentrate on the Majorana case. Writing, $\sin^2(\tilde{p}k/2) = \frac{1}{2}(1-\cos(\tilde{p}k))$, we can separate out the contributions of the planar and non-planar parts to the polarization tensor. Making use of the Feynman parameterization, one obtains,

$$i\Pi^{\mu\nu}(p) = -2g^2 \int_0^1 dx \int \frac{d^3l}{(2\pi)^3} \left[\frac{1}{(l^2+M^2)^2} - \frac{1}{2}\frac{(e^{\tilde{p}l} + e^{-\tilde{p}l})}{(l^2+M^2)^2}\right] M^{\mu\nu}\,, \tag{2.10}$$

with, $M^2 \equiv -x(x-1)p^2 + m^2$ and

$$\begin{aligned} M^{\mu\nu} = 2\Big[&2l^\mu l^\nu + (2x-1)(l^\mu p^\nu + l^\nu p^\mu) - \eta^{\mu\nu}(l^2 + (2x-1)l\cdot p) + \\ &+ 2x(x-1)p^\mu p^\nu - \eta^{\mu\nu}x(x-1)p^2 - m^2\eta^{\mu\nu} + mp^\alpha\epsilon^{\mu\nu\alpha}\Big]\,. \end{aligned}$$

The planar part can be readily evaluated, making use of the Pauli-Villars regularization; for the purpose of comparison, it is better to evaluate both the planar and nonplanar parts in the same regularization scheme, as will be done below. Exponentiating the denominator using the familiar Schwinger parameterization: $1/k^2 = \int_0^\infty d\alpha \exp(-\alpha k^2)$, we get,

$$i\Pi^{\mu\nu}(p) = -2g^2 \int dx \int d\alpha \int \frac{d^3l}{(2\pi)^3} 2\left[M_P^{\mu\nu} - M_{NP}^{\mu\nu}\cdot e^{\frac{-\tilde{p}^2}{4\alpha}}\right] \alpha\cdot\exp\alpha(-l^2 - M^2)\,,$$

wherein,

$$M_P^{\mu\nu} = (2l^\mu l^\nu - \eta^{\mu\nu}l^2) + x(x-1)(2p^\mu p^\nu - \eta^{\mu\nu}p^2) - m^2\eta^{\mu\nu} + mp^\alpha\epsilon^{\mu\nu\alpha}\,. \tag{2.11}$$

Below, we give the components of the non-planar tensor part, which has additional terms:

$$M_{NP}^{\mu\nu} = M_P^{\mu\nu} + M_n^{\mu\nu}\,; \tag{2.12}$$

the components of the additional tensor part turn out to be:

$$M_n^{00} = -\frac{\tilde{p}^2}{4\alpha^2}\,, \qquad M_n^{11} = \frac{\tilde{p}^2}{4\alpha^2} - \frac{\tilde{p}_2^2}{2\alpha^2}\,, \qquad M_n^{22} = \frac{\tilde{p}^2}{4\alpha^2} - \frac{\tilde{p}_1^2}{2\alpha^2} \tag{2.13}$$

$$M_n^{01} = M_n^{02} = 0\,, \qquad \text{and} \qquad M_n^{12} = \frac{\tilde{p}_1\tilde{p}_2}{2\alpha^2}\,. \tag{2.14}$$

The extra terms in the nonplanar part appear due to the momentum redefinition.

2.2 The odd parity sector

The parity odd part of the vacuum polarization tensor gives rise to the induced CS term. Carrying out the momentum integration, one finds,

$$i\Pi^{\mu\nu}_{\text{odd}}(p) = -\frac{mg^2}{2\pi^{\frac{3}{2}}} p^\alpha \epsilon^{\mu\nu\alpha} \int dx \int \frac{d\alpha}{\sqrt{\alpha}} \left[e^{-\alpha M^2} - e^{-\alpha M^2 - \frac{\tilde{p}^2}{4\alpha}} \right]. \tag{2.15}$$

The integrals are finite and the α integration can be done straightforwardly:

$$i\Pi^{\mu\nu}_{\text{odd}}(p) = -\frac{mg^2}{2\pi^{\frac{3}{2}}} p^\alpha \epsilon^{\mu\nu\alpha} \int dx \left[\frac{\sqrt{\pi}}{M} - \sqrt{\frac{2\tilde{p}}{M}}\, K_{1/2}(\tilde{p}\, M) \right], \tag{2.16}$$

where, $K_{1/2}(\tilde{p}\, M)$ is the modified Bessel function of the second kind. Using,

$$K_{1/2}(z) = \sqrt{\frac{\pi}{2z}}\, e^{-z}\,, \tag{2.17}$$

we end up with,

$$i\Pi^{\mu\nu}_{\text{odd}}(p) = -\frac{mg^2}{2\pi} p^\alpha \epsilon^{\mu\nu\alpha} \int dx \left[\frac{1}{M} - \frac{1}{M}\, e^{-\tilde{p}\, M} \right], \tag{2.18}$$

as has been noticed earlier [30]. In the lowest order in p, the planar part gives rise to the the first term in the following CS term, in the effective action:

$$W[A] = \frac{m}{|m|} \frac{g^2}{4\pi} \int d^3x \left[\epsilon^{\mu\nu\lambda} \left(A_\mu \partial_\nu A_\lambda + \frac{2i}{3} g A_\mu * A_\nu * A_\lambda \right) \right]. \tag{2.19}$$

Notice that the cubic term in the above expression can be derived by calculating the 3-point function, where the net phase factor at the vertices does not vanish and depends only on the external momentum, accounting for the star product.

There exists the following two physically interesting limits:

In the limit $\theta \to 0$, the planar and non-planar contributions cancel each other. This is expected, since the Majorana fermions do not couple to the photons in the ordinary theory. Hence, this turns out to be the ordinary theory limit.

The limit $m \to \infty$ keeping θ constant. This situation arises, as will be seen later, when the regulator field is introduced. It is this limit in the ordinary theory that generates the CS term. Here, we see that, the non-planar contribution vanishes leaving the first term as the induced CS term. The large m limit also corresponds to the extreme noncommutativity limit, in which, all the non-planar contributions vanish and one is only left with the planar diagrams.

The observation that the above two limits, $\theta \to 0$ and $M \to \infty$ do not commute has been termed as the discontinuity in θ [30].

2.3 Parity even part

Unlike the parity odd case, it can be easily seen that the momentum integration here, gives rise to a divergent contribution, needing regularization. The main purpose being the identification of UV/IR mixing, we focus on the trace of the self-energy. Using the identities given earlier and carrying out the l and α integrations, we obtain (see appendix),

$$\begin{aligned} i\Pi^{\mu\mu}_{\text{even}}(p) = g^2 \int dx \Bigg[& \left\{ \frac{3\Lambda}{2\pi^2} e^{-\frac{M}{\Lambda}} + \frac{1}{2M\pi^2}(-x(1-x)p^2 + 3m^2) \right\} - \\ & - \left\{ \frac{3\Lambda_{\text{eff}}}{2\pi^2} e^{-\frac{M}{\Lambda_{\text{eff}}}} + \frac{1}{2M\pi^2}(-x(1-x)p^2 + 3m^2)e^{-\tilde{p}M} \right\} + \\ & + \left\{ \frac{M}{2\pi^2}\Lambda^2_{\text{eff}} \left(1 + \frac{\Lambda_{\text{eff}}}{M}\right) \tilde{p}^2 \right\} \Bigg], \end{aligned} \tag{2.20}$$

where a cut-off parameter Λ has been introduced to take care of the small α divergence. Here, $\Lambda^{-2}_{\text{eff}} = \tilde{p}^2 + \frac{1}{\Lambda^2}$ is the effective cut-off. In the above expression, the first term represents the planar contribution, where, the divergent and the gauge invariant pieces have been isolated. The second and third terms are the non-planar contributions.

Notice that even in the absence of the cut-off Λ, the non-planar contribution receives a natural cut-off due to the non-commutativity paramater, which makes the integrals UV finite. Like the odd parity case, various limits yield a number of interesting behaviors.

$\tilde{p} \ll 1/\Lambda$, corresponds to taking the $\theta \to 0$ limit, and $\Lambda_{\text{eff}} \equiv \Lambda$. We see that dropping the θ dependent terms, the planar and non-planar contributions cancel each other, reproducing the result of the ordinary theory.

$\tilde{p} \gg 1/\Lambda$, corresponds to the limit $\Lambda \to \infty$, with $\Lambda^{-1}_{\text{eff}} \equiv \tilde{p}$ and we end up with a piece $\frac{2}{\tilde{p}}\times$ finite, where we have neglected a divergent part in the planar piece, which is exactly cancelled by the regulator field. We see that the limit $\tilde{p} \to 0$ is IR singular. Although, θ regulates the integrals by providing a UV cut-off, it produces a new IR singularity. This is the UV/IR mixing, a characteristic feature of the noncommutative theories [5]. The physical implications of the above IR singularity need further study. This is of interest for theories dealing with LLL physics and other cases involving phase transitions in 2+1 dimentions. It would be interesting to look at the above scenerio at finite temperature, as it is known from the ordinary theories that, temperature effects often bring in IR divergences as well [40]. However, we shall confine ourself here to the study of finite temperature effects on the parity odd part of the polarization tensor, leaving the above analysis for a future work.

3. Finite temperature effects on induced Chern-Simons term

In this section, we explore the effect of finite temperature on the one-loop parity odd part of the polarization tensor. We take recourse to the widely used method in thermal field theory, the Imaginary time formalism for studying the equilibrium systems, where the time variable is given up in favour of temperature [40, 41].

In ordinary theories, it is known that some amplitudes at finite temperature are non-analytic functions of the argument p, i.e., the quantities $\Pi(p_0 = 0, \overline{p} \to 0)$ and $\Pi(p_0 \to 0, \overline{p} = 0)$ do not yield identical results. Here, we would be interested in calculating the odd parity part only in the static limit ($p_0 = 0$, $\overline{p} \to 0$), as in the other limit, the effect of noncommutativity is absent, to start with.

Just like in the ordinary theory, we shall take over the expressions at zero temperature, and replace the continuous energy variable k_3 by $2\pi n/\beta$ for bosons and $\frac{2\pi}{\beta}(n+\frac{1}{2})$ for fermions, and the integration over this variable is replaced by a discrete sum: $\int (dk_3/2\pi) \to \frac{1}{\beta}\sum_n$. Hence, the vacuum polarization tensor in the static limit for the fermions in the adjoint representation at finite temperature is:

$$i\Pi^{\mu\nu}_{(\beta,\mu)}(0,\overline{p}) = -\frac{4g^2}{\beta}\sum_n \int \frac{d^2k}{(2\pi)^2}\frac{M^{\mu\nu}\cdot \sin^2(\tilde{p}k/2)}{((k-p)^2+m^2+\omega_n^2)(k^2+m^2+\omega_n^2)}\,. \tag{3.1}$$

At finite temperature, the most general expression for the polarization tensor can be written as,

$$\Pi^{\mu\nu} = \Pi^{\mu\nu}_{\text{even}}(p) - \epsilon^{\mu\nu\alpha}p^\alpha \Pi^{\text{odd}}(p)\,. \tag{3.2}$$

We would be interested in calculating the one-loop contribution to the parity odd part, $\Pi_{\text{odd}}(p)$, which gives rise to the induced CS term. It is convenient to do the summation first, in order to get an expression in a closed form. Hence, after Feynman parameterization, we have the planar and non-planar pieces isolated as,

$$\Pi_{\text{odd}}(p) = -\frac{4mg^2}{\beta}\sum_{n=-\infty}^{\infty}\int_0^1 dx \int \frac{d^2l}{(2\pi)^2}\left[\frac{1-\frac{1}{2}\{e^{il\tilde{p}}+e^{-il\tilde{p}}\}}{(M_1^2+((2n+1)\frac{\pi}{\beta}+i\mu)^2}\right], \tag{3.3}$$

where $M_1^2 = \overline{l}^2 - x(1-x)p^2 + m^2$ and μ is the chemical potential. The summation can be done by standard methods. We note that, both the exponentials in (3.3) contribute the same. Going to polar coordinates and setting $p_1 = 0$ for convenience, we are left with,

$$\begin{aligned}\Pi_{\text{odd}}(p) \;=\; & \frac{-ig^2}{2\pi^2}\frac{\partial}{\partial m}\int_0^1 dx\int_0^{2\pi}\int_0^\infty \frac{rdrd\phi}{(r^2+M^2)^{\frac{1}{2}}}\left[2 - \frac{2}{e^{\beta\sqrt{r^2+M^2}+\mu}+1} - \right.\\ & \left. - \frac{2}{e^{\beta\sqrt{r^2+M^2}-\mu}+1}\right][1-e^{ir\tilde{p}_2\cos\phi}]\,,\end{aligned}$$

where as before, we have defined, $M^2 = -x(1-x)p_2^2 + m^2$. The calculation of the planar part is straightforward, yielding the following temperature dependence for the coefficient of the CS term for the case of $\mu = 0$,

$$\Pi^P_{\text{odd}}(0) = \frac{2ig^2}{\pi}\frac{m}{|m|}\tanh\left(\frac{\beta m}{2}\right). \tag{3.4}$$

This is the same result as one gets in the ordinary theory with fundamental fermions. It is worth noting that, the above result can be generalized to selective field configurations, where the full effective action depends on temperature in a manner, that preserves the large gauge invariance [43].

The result of integration for the nonplanar piece using the identities given in the appendix is,

$$\Pi^{NP}_{\rm odd}(p) = -\frac{2ig^2}{\pi}\int_0^1 dx\, \frac{m}{|M|}\left[e^{-M\tilde{p}_2} - 2\left\{2e^{-M\sqrt{\beta^2+\tilde{p}_2^2}} - \sum_{n=1}^{\infty} e^{-M\sqrt{n^2\beta^2+\tilde{p}_2^2}}\right\}\right], \qquad (3.5)$$

wherein, the summation over n in the above expression appears upon using the result, $\frac{1}{e^x+1} = 2e^{-x} - \sum_{n=1}^{\infty} e^{-nx}$. At this stage, we can check that the limit $\theta \to 0$ is smooth, since,

$$\Pi^{NP}_{\rm odd}(0) = -\frac{2ig^2}{\pi}\frac{m}{|m|}\tanh\left(\frac{\beta m}{2}\right), \qquad (3.6)$$

which is exactly the negative of (3.4). As is obvious, the planar and the non-planar parts cancel even at finite temperature. Also, just like at zero temperature, the nonplanar contribution vanishes in the large m limit. Hence, the discontinuous behaviour of θ persists even at finite temperature.

Since, at finite temperature we have an extra dimensionful parameter β, two different regimes of high temperature can be studied: $p\theta/\beta \ll 1$, and $p\theta/\beta \gg 1$ similar to the limits considered by [44, 45, 47]. We also find that the limit $p\theta/\beta \ll 1$ produces the results of the conventional field theory. In our case, this limit corresponds to taking $\theta \to 0$, keeping m fixed, which, as we have checked, indeed reproduces the ordinary theory results.

The other regime of high temperature, $p\theta/\beta \gg 1$, where the thermal wavelength is much smaller than the noncommutativity scale shows some interesting features. To find out the behavior of the nonplanar part in this regime, we need to perform the remaining summation in (3.5), which can be done using the Poisson summation formula:

$$\sum_{n=1}^{\infty} e^{-M\sqrt{n^2\beta^2+\tilde{p}_2^2}} = -\frac{1}{2}e^{-M\tilde{p}_2} + \sqrt{2\pi}\left[\frac{1}{2}F(0) + \sum_{n=1}^{\infty} F(2\pi n)\right], \qquad (3.7)$$

with the function $F(2\pi n)$ given by,

$$\begin{aligned} F(2\pi n) &= \sqrt{\frac{2}{\pi}}\int_0^\infty dt e^{-M\sqrt{t^2\beta^2+\tilde{p}_2^2}}\cos(2\pi n t) \\ &= -\sqrt{\frac{2}{\pi}}\frac{1}{\beta}\frac{\partial}{\partial M}K_0\left(\frac{\tilde{p}_2}{\beta}\sqrt{4\pi^2 n^2 + M^2\beta^2}\right). \end{aligned}$$

Here, we have made use of the result [42],

$$\int_0^\infty \frac{dx}{\sqrt{b^2+x^2}} e^{-a\sqrt{x^2+b^2}}\cos(tx) = K_0(b\sqrt{t^2+a^2}),$$

K_0 being the modified Bessel's function of the second kind. Now, we are left with the summation of $F(2\pi n)$ which seems problematic due to the presence of a squareroot in the expression. However, using the following integral representation for K_0,

$$K_0(z) = \frac{1}{2}\int_0^\infty \frac{dt}{t} e^{-\frac{1}{2}(t+z^2/t)}, \qquad (3.8)$$

it is possible to get rid of the squareroot, converting the summation in to a gaussian form. Note that the change of order of summation and integration is not justified at $z = 0$, where the above integral blows up. Thus, we

$$\sum_{n=1}^{\infty} F(2\pi n) = -\sqrt{\frac{2}{\pi}}\frac{1}{2\beta}\frac{\partial}{\partial M}\int_0^{\infty}\frac{dt}{t}e^{-\frac{1}{2}\left(t+\frac{M^2\tilde{p}_2^2}{t}\right)}\sum_{n=1}^{\infty} e^{-\frac{2\tilde{p}_2^2\pi^2}{\beta^2 t}n^2}\,. \tag{3.9}$$

In the limit $p\theta/\beta \gg 1$, the temperature dependent terms in the above equation are exponentially damped. The above can be evaluated term by term. Considering the leading order contribution, we get:

$$\sum_{n=1}^{\infty} F(2\pi n) \approx \frac{T\sqrt{M^2\tilde{p}_2}}{(M^2+(2\pi T)^2)^{3/4}}e^{-\tilde{p}_2\sqrt{M^2+(2\pi T)2}}\,. \tag{3.10}$$

Using (3.10), the summation in (3.7) can be written as,

$$\sum_{n=1}^{\infty} e^{-M\sqrt{n^2\beta^2+\tilde{p}_2^2}} \approx -\frac{1}{2}e^{M\tilde{p}_2} + \frac{\tilde{p}_2}{\beta}K_1(M\tilde{p}_2) + \frac{T\sqrt{M^2\tilde{p}_2}}{(M^2+(2\pi T)^2)^{3/4}}e^{-\tilde{p}_2\sqrt{M^2+(2\pi T)2}}\,. \tag{3.11}$$

Using (3.11) in (3.5), the nonplanar contribution to the parity odd part for the regime $p\theta/\beta \gg 1$ comes out to be:

$$\Pi_{\rm odd}(p) \approx -\frac{4ig^2}{\pi}\int_0^1 dx\frac{m}{|M|}\left[\tilde{p}_2 T K_1(\tilde{p}_2 M) - 2e^{-M\tilde{p}_2} + \right. \\ \left. + \frac{T\sqrt{2\pi M^2\tilde{p}_2}}{(M^2+(2\pi T)^2)^{3/4}}e^{-\tilde{p}_2\sqrt{M^2+(2\pi T)2}}\right]\,. \tag{3.12}$$

We note that the leading order behavior of the above expression is linear in T. The nonplanar piece shows a peculiar high temperature behavior, very different from the planar piece, which goes as $1/T$. The linear T dependence of the non-planar piece is similar to the behavior of bosonic loops in ordinary theories at finite temperature, in the static limit [46].

It is interesting that for thermal wavelengths larger than the noncommutativity scale, the nonplanar contribution to the odd parity part of the polarization tensor shows the temperature dependence characteristic of fermions and for wavelengths smaller than the θ scale, a temperature dependence characteristic of bosons. Evidence from other works [47], regarding these two regimes of high temperature for the nonplanar part suggests that, the system undergoes some kind of a phase change once the temperature is raised beyond $\sqrt{\theta}$. Clearly, the regime $p\theta/\beta \gg 1$ needs a better understanding.

Also, the above result for the non-planar piece is not large gauge invariant. In the case of ordinary theory, the full effective action for selective, time independent backgrounds has been shown to be invariant under large gauge transformations at finite temperature [48]. Hence, it is worthwhile to perform a similar non-perturbative computation in this context as well.

4. Conclusions

To conclude, the CS term induced from the fermions in the fundamental representation does not receive any corrections due to the noncommutativity of space, both at zero and finite temperature, at one loop. However, the CS term induced from the adjoint fermions at zero temperature, shows a discontinuous behavior as a function of the noncommutativity parameter, which persists even at finite temperature. The parity even part shows UV/IR mixing. There are two regimes of high temperature one can distinguish. The regime $p\theta/\beta \ll 1$, produces the results of ordinary theory, whereas in the regime $p\theta/\beta \gg 1$, the nonplanar contribution to the odd parity part of the polarization tensor shows a very different temperature dependence; the behavior is similar to that of bosonic loops in ordinary theories, whereas the planar piece still goes as $1/T$, a feature characteristic of the behavior of fermionic loops. Evidently, the contrasting high temperature behaviors of the planar and non-planar pieces in the above regime need further study.

The computation of the effective action invariant under large gauge transformations, should throw more light on this aspect. Also, it would be worthwhile to look at the even parity part in the above two regimes of high temeprature as would be the study of the polarization tensor at finite density, even for the non-abelian case. These works are in progress and we hope to get back to these issues in future.

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A. Formulae for relevant integrals

1. To do the three dimensional integrals of the nonplanar piece, the following identities in polar coordinates are useful. The angular integration can be done using,

$$\int_0^\pi d\phi e^{ir|p|\cos(\phi)} \sin^{n-2}(\phi) = \frac{\Gamma(\frac{n-2}{2})\sqrt{\pi}}{(\frac{r|p|}{2}^{\frac{n-2}{2}})} J_{\frac{n-2}{2}}(r|p|)\,, \tag{A.1}$$

where $J_{\frac{n-2}{2}}$ is the Bessel's function of order $\frac{n-2}{2}$. For the temperature independent part of the nonplanar piece, the integration of the radial part can be done through,

$$\int_0^\infty dr r^{\frac{n}{2}} (c^2+r^2)^m J_{\frac{n-2}{2}}(r|p|) = \left(\frac{2}{|p|}\right)^{m+1} \frac{c^{\frac{n}{2}+m}}{\Gamma(-m)} K_{\frac{n}{2}+m}(c|p|)\,, \tag{A.2}$$

whereas for the temperature dependent part, we use,

$$\int_0^\infty dr r J_0(pr) \frac{e^{-a\sqrt{r^2+M^2}}}{\sqrt{r^2+M^2}} = \frac{e^{-M\sqrt{p^2+a^2}}}{\sqrt{p^2+a^2}}\,. \tag{A.3}$$

2. The following properties and identities involving the modified Bessel's function of the second kind, $K_\nu(z)$ have been used in the text:

   (a)
$$\int_0^\infty dx x^{\nu-1} e^{\frac{\beta}{x}-\gamma x} = 2\left(\frac{\beta}{\gamma}\right)^{\frac{\nu}{2}} K_\nu(2\sqrt{\beta\gamma})\,, \tag{A.4}$$

   (b)
$$\begin{aligned} K_{\pm\frac{1}{2}}(z) &= \sqrt{\frac{\pi}{2z}}e^{-z}\,, \\ K_{\pm\frac{3}{2}}(z) &= \sqrt{\frac{\pi}{2z}}\left(1+\frac{1}{z}\right)e^{-z}\,, \end{aligned}$$

   (c)
$$\frac{d}{dz}K_0(z) = -K_1(z)\,. \tag{A.5}$$

   (d) For small arguments, i.e., $z \to 0$
$$K_1(z) \approx \frac{1}{z}\,, \tag{A.6}$$

   and for large arguments, i.e., the asymptotic expansion to leading order is,
$$K_1(z) \approx \sqrt{\frac{\pi}{2z}}e^{-z}\,. \tag{A.7}$$

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